

Classe: TS2ET	Date: 8 décembre 2020	Type Devoir surveillé
Devoir n°3		
Thème: Transformée de Laplace		

Exercice 1

A l'aide du formulaire, déterminer les transformées de Laplace des fonctions causales suivantes:

$$f_1(t) = 2e^{4t}U(t)$$

$$f_2(t) = (3t - 5)U(t)$$

$$f_3(t) = (t^4 - e^{-3t})U(t)$$

$$f_4(t) = 3\cos(7t) \cdot U(t)$$

$$f_5(t) = t^3 e^{-3t}U(t)$$

$$f_6(t) = e^{3t} \sin(t)U(t)$$

Exercice 2

A l'aide du formulaire, déterminer les originaux des fonctions suivantes:

$$1) F(p) = \frac{1}{p} + \frac{3}{p^2}$$

$$2) F(p) = \frac{2}{p} + \frac{3}{p+1}$$

$$3) F(p) = \frac{1}{p} e^{-2p}$$

$$4) F(p) = \frac{2}{p} - \frac{3}{2p^2} + \frac{2}{p^2+4}$$

$$5) F(p) = \frac{6}{p^2+9}$$

$$6) F(p) = \frac{3p}{p^2+25}$$

$$7) F(p) = \frac{1}{(p+3)^2}$$

Conexion du devoir

Exercice 1 : • $f_1(t) = 2e^{4t}U(t)$ donc $F_1(p) = \mathcal{L}(2e^{4t}U(t)) = 2 \times \frac{1}{p-4}$

$$F_1(p) = \frac{2}{p-4}$$

(1)

• $f_2(t) = (3t-5)U(t) = 3tU(t) - 5U(t)$

donc $F_2(p) = 3\mathcal{L}(tU(t)) - 5\mathcal{L}(U(t)) = 3 \times \frac{1}{p^2} - 5 \times \frac{1}{p}$

$$F_2(p) = \frac{3}{p^2} - \frac{5}{p}$$

(2)

• $f_3(t) = (t^4 - e^{-3t})U(t) = t^4U(t) - e^{-3t}U(t)$

donc $F_3(p) = \mathcal{L}(t^4U(t)) - \mathcal{L}(e^{-3t}U(t)) = \frac{4!}{p^5} - \frac{1}{p+3}$

$$F_3(p) = \frac{24}{p^5} - \frac{1}{p+3}$$

(2)

• $f_4(t) = 3\cos(7t)U(t)$

donc $F_4(p) = 3\mathcal{L}(\cos(7t)U(t)) = 3 \times \frac{p}{p^2+49}$

$$F_4(p) = \frac{3p}{p^2+49}$$

(1)

• $f_5(t) = t^3 e^{-3t}U(t)$ donc

$$F_5(p) = \frac{3!}{(p+3)^4} = \frac{6}{(p+3)^4}$$

(2)

$$f_6(t) = e^{3t} \sin(t) U(t) \quad \text{donc} \quad F_6(p) = \frac{1}{(p-3)^2 + 1} \quad (2)$$

Exercice 2 1°) $F(p) = \frac{1}{p} + \frac{3}{p^2}$ donc $f(t) = U(t) + 3tU(t) = (1+3t)U(t) \quad (1)$

2°) $F(p) = \frac{2}{p} + \frac{3}{p+1}$ donc $f(t) = 2U(t) + 3e^{-t}U(t) = (2+3e^{-t})U(t) \quad (1)$

3°) $F(p) = \frac{1}{p} e^{-2p}$ donc $f(t) = U(t-2) \quad (2)$

4°) $F(p) = \frac{2}{p} - \frac{3}{2p^2} + \frac{2}{p^2+4}$ donc $f(t) = 2U(t) - \frac{3}{2}tU(t) + \sin(2t)U(t) = \left(2 - \frac{3}{2}t + \sin(2t)\right)U(t) \quad (2)$

5°) $F(p) = \frac{6}{p^2+9} = 2 \times \frac{3}{p^2+3^2}$ donc $f(t) = 2 \cdot \sin(3t)U(t) \quad (1)$

6°) $F(p) = \frac{3p}{p^2+25} = 3 \frac{p}{p^2+5^2}$ donc $f(t) = 3 \cdot \cos(5t)U(t) \quad (1)$

7°) $F(p) = \frac{1}{(p+3)^2}$

donc $f(t) = tU(t)e^{-3t} = te^{-3t}U(t) \quad (2)$